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Linear Control Systems

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Florin Stratulat **Florin Ionescu**

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THEORY AND APPLICATIONS

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PREFACE

This work presents the usual contents of the linear systems theory, centered mainly on the bases of the control systems and also on their systematic interpretation.
In elaborating this work, the authors had in view the accomplishment of three desiderates required by the efficiency of the instructive process:

- the ensuring of a defining and procedural frame able to stimulate a preparation in large profile and also adaptable at the actual informatic dynamics;
- pointing out only those notions and methods that really stimulate the creativity of the student, avoiding the abuse of inutile information;
- insisting on applications that effectively contribute to the development of the skills and judgment that allow the future specialist to solve the complex problems imposed by the practice in the context of the permanent scientific and technologic progress.

On the basis of those desiderates, this work has a development that covers, with the amendments of a easy assimilation and with an adequate share, both the engineer methods that has become traditional in the study of the linear control systems (such as those in frequency) and the modern ones indispensable to the analysis and synthesis assisted by computer (by using the program packages MATLAB-Simulink, MATCAD, AutoCAD).
Chapter 1 presents the concept of system and its properties starting from the notion of process, the last one being considered from the point of view of the technologic functionality. The functional equations are determined (the mathematical model) and the transfer functions corresponding to a very large scale of industrial processes.
Chapter 2 introduces the concept of realization (on state) for a rational strict proper $H(\lambda)$, the notion of equivalence and the **eliminant** of two polynomials. On the basis of the **Hankel matrix** associated to the rational $H(\lambda)$, an algorithm of construction of the minimal realization is presented.
Chapter 3 points out some particularities of the forced response of the linear systems to the so-called standard input sizes: **polynomial** and **harmonics**. Each application is analytically solved by at least one single numerical method (in real time or by operational calculus), both for continuous linear systems and for the discreet ones. Finally, the analytical solution is validated by computer simulation, being presented the MATLAB program for the determination and for the graphic representation of the response of the given system (by pointing out also the details regarding the graphic **personalization**).

Chapter 4 presents the procedure of **discretization** of a time continuous function and of a continuous linear system respectively.

Chapter 5 points out the representations in frequency of the transfer function: **the semilogarithmic frequency characteristics and the transfer locus (the Nyquist locus).**

For the accommodation of the student with the manual drawing of the semilogarithmic frequency characteristics, specific diagrams are presented (in the end of this work).

For the validation of the graphic representations, the MATLAB programs for the determination and for the graphic representation of the **response in frequency** (the amplitude-pulsation and phase-pulsation diagrams, the Bode diagrams or the Nyquist locus) are presented in detail.

Chapter 6 presents the three possibilities of the linear system connection:
series, parallel and **reaction** and their properties.

Chapter 7 presents the concept of the **stability** of a linear control system and different criteria for the stability analysis: **Nyquist, Hurwitz, Bode, Mihailov**.

For the validation of the stability, the MATLAB programs for the determination and graphic representation of the **step response** are presented in detail.

Chapter 8 introduces the concept of the **root locus** for a control system and presents the algorithm of the root locus drawing and his use in the analysis and synthesis of the linear control systems.

Chapter 9 introduces the structural concepts of the linear systems (controllability and observability, and structural decomposition). The state estimator theory, systems stabilization problem, and design algorithm of the dynamic stabilization compensator are presented.

The annexes 1 and 2 present the Laplace and Z operators, their properties and theorems as well as the Laplace and Z transform tables.

The annex 3 introduces general notions regarding the graphic representations in MATLAB and describes a part of the MATLAB functions from the CONTROL SYSTEM DESIGN toolbox.

The annex 4 presents the modeling and simulation of the systems.

The theoretic presentation is proposed under the form of a SUMMARY that covers only enunciation, problem formulations, theorems, commentaries, algorithms and conclusions (avoiding the mathematical demonstrations), and the numerous examples facilitate the reader the understanding of the subtleties of the different concepts and methods.

This work is dedicated to the students who prepare themselves in the domain of the automatics and of controlling by computer the industrial processes, to the specialists from the domain of the automatic control, and as well to the ones from other domains of activity that attend the courses of *System Theory* or *Industrial Automatics*.

The Authors

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CHAPTER I
MATHEMATICAL MODEL OF THE INDUSTRIAL INSTALLATIONS

1.1. Summary

The elementary systemic representation of a linear industrial process is given by the relations:

$$\begin{cases} \dot{x}(t) = A \cdot x(t) + B \cdot u(t) + E \cdot v(t) \\ y(t) = C \cdot x(t) \\ z(t) = D \cdot x(t) \end{cases} \tag{1.1}$$

where: $x(t) \in R^n$ represents the state, $u(t) \in R^m$ - command, $y(t) \in R^p$ - measured output, $z(t) \in R^q$ - quality, $v(t) \in R^r$ - perturbation. They are the vectors of some linear spaces (Euclidean) dimensional finite, and $A \in R^{nxn}$, $B \in R^{n\,x\,m}$, $C \in R^{p\,x\,n}$, $D \in R^{q\,x\,n}$, $E \in R^{n\,x\,r}$ are constant matrix.
The interpretation is presented in Fig. 1.1.

Definition 1.1. A triplet (A, B, C), which is explained by the following relations, is named linear system:

$$\begin{cases} x' = A \cdot x + B \cdot u, x(o) = x_o \\ y = C \cdot x \end{cases}, \tag{1.2}$$

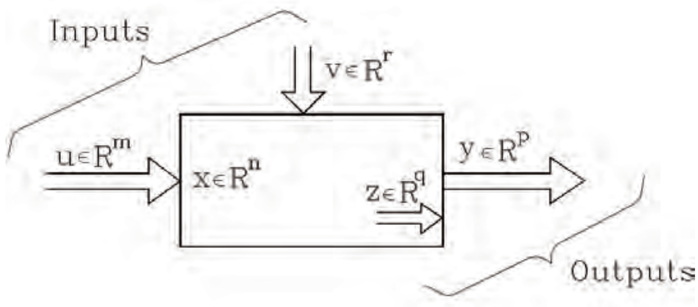

Fig. 1.1